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International Journal of Applied Mathematics and Statistics™
ISSN: 0973-7545 (Online), ISSN 0973-1377 (Print)
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2015, Volume 53, Issue Number: 6

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Modeling Dependence of Asian Stock Markets Using Dynamic Copula Functions

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ABSTRACT

The aim of this paper is to analyze the time dynamic of the dependence structure between Jakarta Stock Exchange Index (JKSE) and four Asian Indexes, Hang Seng, Nikkei, KOSPI, and Straits Times Index (STI). We set up the dynamic of the dependence structure based on Patton's formulation. In addition, we observe the effect of tail dependence of different copula functions on VaR estimation. We use AR(1)-GJR(1,1) to fit the margin of each series. Then we apply time-varying Normal Copula, time-varying Rotated Gumbel Copula (RGC), and time-varying Symmetrized-Joe-Clayton Copula (SJC) to model the dependence between the stock indexes. Next, we look for the best copula representing the relation among stock indexes. To do this we apply AIC, BIC, and Log-Likelihood to choose the most representing copula. Our result shows that time-varying Normal and SJC are best fit the pair JKSE-STI, while Normal copula fits better the rest of the pairs. We also calculate VaR and CVaR from the fitted copulas, and conclude that VaR using Normal copula is more aggressive than RGC and SJC. This means that Normal copula gives higher opportunities to investors to gain higher return as they set at a higher risk. Finally, backtesting is carried out to test the accuracy of VaR and CVaR models. The test shows that Normal copula gives lower exceedance than RGC and SJC copula for all portfolios.

Keywords: dependence structure, dynamic copula, value-at-risk, conditional value-at-risk

Mathematics Subject Classification: 62H20, 62G32

Journal of Economic Literature (JEL) Classification : G11, G15, G17

1. INTRODUCTION

Understanding the structures of dependence among data of financial time series is the most important step in modeling multivariate distribution. It describes the relationship among the random variables that represent the market indexes. Studies on dependence structure of financial markets are still a challenge for researchers or practitioners in financial area, see for examples in \([2]\), or \([3]\). The most popular approach to model the structure of dependence is the multivariate normal model. This approach assumes that the distributions are normal multivariate and the parameters are estimated by applying the method of mean-variance. As mentioned in many literatures, such as \([4]\), \([5]\), and \([6]\), that the univariate normal distribution is insufficient model for modeling real assets. To deal with this, one should find another method which is more flexible and can solve the problem mentioned above.

The most recent method to treat multivariate distribution appears in financial analysis is copula function. This approach is based on separating the multivariate distribution from dependence relationship into its univariate distributions. A copula is a function that connects the univariate marginal distributions into the multivariate distribution. Several families of copula functions already
discussed in the literature, such as in [7] and [8] with varieties of models of dependence structure. The recent articles discuss copula functions for modeling the dependence structure in financial markets are available in [8].

In recent years, the attention of researchers or practitioners directs to a more dynamic model. The assumption that correlations between asset returns are constant through time has broken the stylized facts, described in [1] or [9]. For this reason, it is unlikely realistic to model the dependence among asset returns based on constant correlation, while it has been found at least, to our knowledge by Patton [1] that correlation is a time-varying coefficient. Furthermore, the extension of Sklar’s theorem to time-varying or conditional distribution is proposed by Patton in [1]. Since then, a large number of publications either the methodology or its application has appeared. Readers who are interested in this growing field of research may refer to [3], [6], [10], [11], [12].

This paper presents the results of a study of time-varying dependence between Indonesian Index (and 4 Asian Indexes. The performance of a range of models, based on three different types of time-varying parameters of copula functions, on a data set of daily returns recorded during the period of 10 June 2010 to 10 June 2014, is assessed and compared. Our study is limited to pair-wise dependence. More specifically, we will use only continuous two-dimensional copulas, Normal, Rotated Gumbel, and SJC (Symmetrized Joe-Clayton) copulas that bind JKSE to 4 other indexes. We also fit each margin with AR(1)-GJR(1,1) using three different density functions

\[ C(u_1, \ldots, u_d) = \text{Pr}(U_1 \leq u_1, \ldots, U_d \leq u_d) \]  

where \( U \) is a random variable with uniform distribution in \([0, 1]\) and \( u \) is the value or the realization of \( U \).

Using this fact, copula can be used to construct multivariate dependence from marginal distributions. Formally, copula can be defined as follows:

**Definition 1** (Nelsen[7] and Embrechts[13]) A d-dimensional copula is a function \( C \) with domain \([0,1]^d\) such that

1. \( C \) is grounded and \( d \)-increasing
2. \( C \) has margin \( C_i \), \( i = 1, \ldots, d \), where \( C_i(u) = u \), \( \forall u \in [0,1]^d \)

The most famous theorem in copula theory is Sklar’s Theorem (1956) which is stated as follows:

**Theorem 1** (Sklar’s Theorem, [7]). Let \( X_1, X_2, \ldots, X_d \) be random variables with distribution functions \( F_1, F_2, \ldots, F_d \), respectively, and joint distribution function \( H \). Then, there exist a function, \( C \) such that

\[ H(x_1, x_2, \ldots, x_d) = C(F_1(x_1), F_2(x_2), \ldots, F_d(x_d)) \]  

for every \( x_1, \ldots, x_d \in \mathbb{R}^d \). If \( F_1, \ldots, F_d \) are all continuous, then \( C \) is unique, otherwise \( C \) is determined only on \( \inf F_1 \times \cdots \times \inf F_d \). Conversely, if \( C \) is a \( d \)-dimensional copula and \( F_1, \ldots, F_d \) are distribution...
functions, then function $H$ as is defined in (2) is a $d$-dimensional distribution function with margins $F_1, \ldots, F_d$.

**Corollary 1 (Nelsen[7])** Let $H$ be a $d$-dimensional distribution function with continuous margins $F_1, \ldots, F_d$ and copula $C$ as in Theorem 1. Hence, for any $u = (u_1, u_2, \ldots, u_d) \in [0,1]^d$, \[
C(u_1, \ldots, u_d) = H(F_1^{-1}(u_1), \ldots, F_d^{-1}(u_d))
\] (3)
where $F^{-1}$ is the generalized inverse.

An important idea by Patton in [1] regarding the dependence structure between markets movement is the change of dependence with respect to time. This is known as time-varying copula. This idea is then extended by Fermanian and Wegkamp in [2] into a model known as pseudo copula. As we consider the dependence in terms of copula functions, then the correlation and their shape will change in time. These two articles have provided two different approaches in applying dynamic copula. Article [1] is to provide dynamical model using copula function and article [2] is to use copula function to represent the dynamic of a system. The definition of time-varying copula of conditional copula is give as follows:

**Definition 2 (Conditional copula)** Let $X$ and $Y$ be random variables. The conditional copula of $(X,Y)$ given $F_{t-1}$, where $X|F_{t-1} \sim F_t$ and $Y|F_{t-1} \sim G_t$, is the conditional joint distribution function of $U_t \equiv F_t(X|F_{t-1})$ and $V_t \equiv G_t(Y|F_{t-1})$, given $F_{t-1}$.

It is very common in time series analysis that a random variable is conditioned on some variables. One often writes the conditioning variables as $F_{t-1}$. In financial area, this variable can be interpreted as lagged returns. It is implicitly described in Definition 2 that multivariate dependence structure can be separated into its univariate margins. To understand how it works, consider the following theorem which is the extension of Theorem 1 proposed by Patton in [1]. For simplicity, it is written in a bivariate distribution.

**Theorem 2 (Sklar’s Theorem for conditional distribution)** Let $F_t$ be the conditional distribution of $X|F_{t-1}$ given the conditioning set $F_{t-1}$. $G_t$ be the conditional distribution of $Y|F_{t-1}$ and $H_t$ be the joint conditional bivariate distribution of $(X,Y|F_{t-1})$. Assume that $F_t$ and $G_t$ are continuous in $X$ and $Y$. Then there exists a unique conditional copula $C_t$ such that \[
H_t(x,y|F_{t-1}) = C_t(F_t(x|F_{t-1}), G_t(y|F_{t-1})|F_{t-1})
\] (4)

Conversely, let $F_t$ and $G_t$ be the conditional distribution of $X_t$ and $Y_t$, and $C_t$ be a conditional copula, then the function $H_t$ defined in (4) is a conditional bivariate distribution function with conditional marginal distributions $F_t$ and $G_t$.

Theorem 2 is Sklar’s theorem for the version of conditional distribution. It is important to note that $F_{t-1}$ must be the same for both marginal distribution and the copula, otherwise $F_t, G_t$ and $H_t$ will not be a joint conditional distribution function [14]. To extract the implied conditional copula from any bivariate conditional distribution, one can apply Sklar’s theorem and using the relation between the distribution and the density function, the bivariate copula density can be derived as follows:
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\[ h_t(x,y|F_{t-1}) = \frac{\partial^2 G_t(x,y|F_{t-1}, F_{t-1})}{\partial x \partial y} \frac{\partial F_t(x|F_{t-1})}{\partial x} \frac{\partial G_t(y|F_{t-1})}{\partial y} = c_t(x,y|F_{t-1}, F_{t-1}) \cdot f_t(x|F_{t-1}) \cdot g_t(y|F_{t-1}) \]
\[ c(u,v|F_{t-1}) = \frac{h_t(x,y|F_{t-1})}{f_t(x|F_{t-1}) \cdot g_t(y|F_{t-1})} \]  \( t \)  \( (5) \)

where \( F_t(x|F_{t-1}) \) and \( v \equiv G_t(y|F_{t-1}) \). Ones may refer to [7] for another class of copulas, which is also known as Archimedean copulas.

3. THE MODEL FOR MARGINS

The marginal distributions that we used to build a joint multivariate distribution are Normal, \( t \)-student, and Skew-\( t \) student. The model for each marginal time series by a general AR(1)-GJR(1,1) model for the continuously compounded returns is given by

\[ r_{it} = c_0 + c_1 r_{it-1} + e_{it} \]  \( (6) \)
\[ e_{it} = \sigma_{it} \varepsilon_{it}, \quad \varepsilon_{it} \sim Skew - t \left( \nu_i, \lambda_i \right) \]  \( (7) \)
\[ \sigma^2_{it} = \omega_i + \alpha_i e_{it-1}^2 + \beta_i \sigma^2_{it-1} + \gamma_i e_{it-1} e_{it-1} \]  \( (8) \)

where \( e_{it} \) and \( e_{it-1} \) are the residuals and one lagged residual of the model \( i \) and the distributions of \( \varepsilon_{it} \) are \( N(\mu_i, \sigma_i), \nu(\nu_i, \lambda_i), \) and Skewed - \( t(\nu_i, \lambda_i) \), where the skewed-\( t \) densities is given by

\[ f(\varepsilon_{it}; \nu, \lambda) = \begin{cases} \frac{bc \left( 1 + \frac{1}{|\nu|} \left( \frac{\nu \varepsilon_{it} + \alpha}{\lambda^2} \right) \right)^{-(\nu+1)/2}}{\Gamma \left( \frac{\nu+1}{2} \right) \sqrt{\pi |\nu|}} & \text{if } \varepsilon_{it} < -a/b \\ \frac{bc \left( 1 + \frac{1}{|\nu|} \left( \frac{\nu \varepsilon_{it} + \alpha}{\lambda^2} \right) \right)^{-(\nu+1)/2}}{\Gamma \left( \frac{\nu+1}{2} \right) \sqrt{\pi |\nu|}} & \text{if } \varepsilon_{it} \geq -a/b \end{cases} \]  \( (9) \)

with constants \( a, b \) and \( c \) defined as

\[ a = 4\lambda c \left( \frac{\nu_i - 2}{\nu_i - 1} \right), \quad b^2 = 1 + 3\lambda_i - a^2, \quad c = \frac{1}{\Gamma \left( \frac{\nu_i + 1}{2} \right) \sqrt{\pi |\nu_i|}} \]

where the parameters \( \nu_i \) and \( \lambda_i \) representing the degrees of freedom and asymmetry, respectively.

4. PARAMETER ESTIMATIONS: INFERENCE FOR MARGIN (IFM)

Rearranging Equation (5) and putting parameters taken into account to the density function, it gives

\[ h_t(x_t, y_t|F_{t-1}; \theta) = f_t(x_t|F_{t-1}; \theta) \cdot g_t(y_t|F_{t-1}; \theta) \cdot c_t(u,v|F_{t-1}; \theta) \]  \( (10) \)

where \( \theta = [\theta_1, \theta_2, \theta_3, \ldots] \) is a vector of parameters of the joint density. Equation (10) suggests that the conditional density function can be decomposed into two problems and estimation will be carried out sequentially; firstly to identify the conditional distribution of the margins for \( X \) and \( Y \) and secondly to establish a functional form for copula \( C \). Thus, the log-likelihood function of Equation (10) is given by

\[ \sum_{t=1}^{T} \log h_t(x_t, y_t|F_{t-1}; \theta) = \sum_{t=1}^{T} \log f_t(x_t|F_{t-1}; \theta) + \sum_{t=1}^{T} \log g_t(y_t|F_{t-1}; \theta) \]
According to the IFM method, the parameters of the marginal distributions are estimated sequentially, in two steps:

1. Estimating the parameters \((\theta_f, \theta_g)\) of the marginal distributions, \(F_t\) and \(G_t\) using maximum likelihood Estimation method (MLE method):

\[
\hat{\theta}_f = \arg \max \sum_{t=2}^{T} \log f_t(x_t | F_{t-1}; \theta_f)
\]

\[
\hat{\theta}_g = \arg \max \sum_{t=2}^{T} \log g_t(y_t | F_{t-1}; \theta_g)
\]

2. Estimating the copula parameter \(\hat{\theta}_c\), given \(\hat{\theta}_f\) and \(\hat{\theta}_g\)

\[
\hat{\theta}_c = \arg \max \sum_{t=2}^{T} \log c_t(x_t, y_t | F_{t-1}; \theta_c)
\]

It is just like the ML estimator method, it verifies the properties of asymptotic normality.

5. TAIL DEPENDENCE

Definitions of tail dependence are mostly related to their bivariate marginal distribution functions. Roughly speaking, tail dependence describes the limiting proportion that one margin exceeds a certain threshold given that the other margin has already exceeded that threshold. Tail dependence captures the behavior of the random variables during extreme events. Informally, in our application, it measures the probability that we will observe an extremely large depreciation (appreciation) of one random variable to another one.

The most common copula family used in financial area comes from elliptical class, that is Gaussian copula or Normal copula and \(t\)-student copula. It is called elliptic because its contour looks like an elliptical form of correlation between the margins. The Pearson correlation coefficient is used to explained the dependence structure which is associated with copula come from elliptical families. The value of this coefficient ranges in interval \([-1, 1]\) and symmetrically distributed.

Archimedean copula is another class of copula. It is very important class of copulas - because of the ease with which they can be constructed and the nice properties they possess. Their dependence measure can vary diversely. Their tail dependence measure may vary from 1 to infinity. For this reason, copula function with specific dependence structure is hardly to compare with different functional forms for copulas. To see the dependence structure among random variables in association with copulas belong to Archimedean class, one may have to assess their tail of the distributions, see [1], [7], or [8].

**Definition 3 (Tail Dependence, [13])** Let \(X\) and \(Y\) be random variables with continuous marginal distribution functions \(F\) and \(G\) and copula \(C\). The coefficient of upper tail dependence of \(X\) and \(Y\) is

\[
+ \sum_{i=1}^{T} \log c_t(x_t, y_t | F_{t-1}; \theta_c)
\]
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\[ r^u = \lim_{u \to 1} P(Y > G^{-1}(u) | X > F^{-1}(u)) = \lim_{u \to 1} \frac{1 - 2u + C(u,u)}{1 - u} \]  
provided that the limit \( r^u \in [0,1] \) and the copula \( C \) exist. If \( r^u \in (0,1) \), \( X \) and \( Y \) are said to be asymptotic all dependent in the upper tail; if \( r^u = 0 \), \( X \) and \( Y \) are said to be asymptotically independent in the upper tail. The coefficient of lower tail-dependence is given by

\[ r^l = \lim_{u \to 0} P(X < F^{-1}(u) | Y < G^{-1}(u)) = \lim_{u \to 0} \frac{C(u,u)}{u} \]  

If \( r^l \in (0,1) \), \( X \) and \( Y \) are said to be asymptotic all dependent in the upper (lower) tail. If \( r^l = 0 \), \( X \) and \( Y \) are said to be asymptotically independent in the lower tail.

6. THE MODEL FOR COPULA

In the following discussion, we follow [1] for the problem setup. The most common copula applied in financial field is Gaussian copula (Normal copula). It is associated with the bivariate normal as follows:

\[ C_{\text{Normal}}(u,v; \rho_1) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho_1^2}} \exp\left(-\frac{s^2 - 2\rho_1 st + t^2}{2(1-\rho_1^2)}\right) ds dt \]  

where \( \Phi^{-1} \) is the inverse of the standard normal c.d.f. and the dependence parameter \( \rho_1 \in (-1,1) \) is the coefficient of linear correlation. Its dynamic equation is

\[ \rho_t = \Lambda \left( \alpha_N + \beta_\theta \rho_{t-1} + \alpha_N \cdot \frac{1}{m} \sum_{j=1}^{m} \Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j}) \right) \]  

where \( \Lambda(x) \equiv (1 - e^{-x})(1 + e^{-x})^{-1} = \tanh(x/2) \) is the modified logistic transformation, designed to keep \( \rho_t \) in \((-1,1)\) and \( m \) is the optimal lag indicated by data returns. Note that in our case, for simplification, we estimate the mean of the margins by \( AR(1) \) \((m = 0)\), instead of \( ARMA(1,10) \) as in [1].

The second copula function used in this paper is the symmetrized Joe–Clayton copula. This copula is a modification of the ‘BB7’ copula [1]. The functional forms for the Joe–Clayton copula is given by

\[ C_{JC}(u,v; \tau^u, \tau^l) = 1 - \left(1 - [(1 - (1 - u)^\kappa]^{-\gamma} + [(1 - (1 - v)^\kappa]^{-\gamma} - 1\right]^{1/\gamma} \]  

where \( \kappa = 1/\log_2(2 - \tau^u) \), \( \gamma = \log_2(\tau^l) \), and \( \tau^u, \tau^l \in (0,1)\).The Joe–Clayton copula has two parameters, \( \tau^u \) and \( \tau^l \), which are measures of dependence known as tail dependence, defined as in (15) and (16). As is stated by Patton in [1], that one weakness of the Joe–Clayton copula is that the existence of a slight asymmetry on their distribution. A more desirable model would have the tail dependence measures completely determining the presence or absence of asymmetry. To this end, Patton [1] propose the ‘symmetrized Joe–Clayton’ copula:

\[ C_{SJC}(u,v; \tau^u, \tau^l) = 0.5 \cdot C_{JC}(u,v; \tau^u, \tau^l) + C_{JC}(1-u,1-v; \tau^u, \tau^l) + u + v - 1 \]  

The symmetrized Joe–Clayton (SJC) copula is only a little modification of the original Joe–Clayton copula with \( \tau^u = \tau^l \).

The third copula function used in this paper is Rotated Gumbel Copula (RGC) (or Survival Gumbel Copula). The RGC has the following form

\[ C_{RGC}(u,v; \theta) = u + v - 1 + C_{G}(1-u,1-v; \theta) \]  

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where $C_{u}$ correspond to the Gumbel copula:

$$C_{u}(u,v;\theta) = \exp\left(-(\log u)^\theta + (-\log v)^\theta\right)^{1/\theta}, \quad \theta \in [1, \infty)$$

(22)

The time dynamics equation for the dependence parameter in RGC, $\theta_t$, is given by

$$\theta_t = \lambda \left( \omega + \beta \theta_{t-1} + \alpha \sum_{i=1}^{10} \left| t_{t-i} - v_{t-i} \right| \right)$$

(23)

where $\lambda(x) = 1 + x^2$ is a polynomial transformation to ensure that $\theta \in [1, \infty)$. The RGC has only lower tail dependence ($t^2 = 0$), which can be obtained by $t^2 = 2 - 2^{-1/\theta}$.

### 7. RISK MEASURE AND BACK TESTING

One of the most popular risk measure is Value at Risk (VaR). It measures the potential loss in value of a risky portfolio over a defined period of time for a given confidence interval. It is often used by commercial and investment banks to capture the potential loss in the value of their portfolios from adverse market movements over a specified period of time which can then be compared to their available cash reserves and capital to ensure that the lost can be covered without putting the firms at risk of default. Suppose a random variable $X$ with continuous distribution function $F$ models losses or negative returns on a certain financial instrument over a certain time horizon. VaR can be defined as

$$P(X_t \leq -\text{VaR}_t(\alpha)) = \alpha \Leftrightarrow \text{VaR}_t(\alpha) = F^{-1}(\alpha)$$

(24)

where $\alpha$ is the confidence level, $F^{-1}$ is the quantile function defined as the inverse of the distribution function. Most financial firms compute a 5% VaR over a one-day holding period. Another important measure of risk is the expected shortfall or the conditional VaR (CVaR) which estimates the potential size of the loss exceeding the VaR. Backtesting is a test applied to VaR to count the number of times that VaR limit has been exceeded by the portfolio returns. The formal statement is defined as follows:

Let $x_{t+1}^i$ denote the realized profit and loss of returns between time $t$ and $t+1$. The “hit” function is defined as

$$l_{t+1}(x) = \begin{cases} 1, & x_{t+1} \leq -\text{VaR}_t \\ 0, & x_{t+1} > -\text{VaR}_t \end{cases}$$

(25)

which is the number of violation of the VaR exceeded by the portfolio returns. To test the significant of the VaR estimate, we use the test discussed in [15]. If the probability of exceeding the VaR is $P_{v} = P \left( X_{t} \leq -\text{VaR}_t(\alpha) \right)$, the test is carried out under the null hypothesis $P_{v} = \alpha$ against the alternative hypothesis $P_{v} \neq \alpha$.

### 8. EMPIRICAL STUDY

In this research, the performance of 5 Asian stock indexes are considered, specially the effect of the Asian Indexes, Hang Seng, Nikkei, KOSPI, Straits Times Index (STI) on the Jakarta Stock Exchange Composite Index (JKSE). The descriptive statistics of the data set of daily log-returns recorded during the period of 10 June 2010 to 10 June 2014 are described in Table 1. This period of time does not witness the fast decrease of world financial markets as a result of the economic crises in 2007-2008.
The five indexes generally exhibited negative skewness and excess kurtosis. Based on the Jarque-Bera test statistics, there is evidence that the log-returns are not normally distributed. The descriptive statistics agree with the stylize facts of financial data as discuss in many literatures. Besides, the means and volatilities are very similar, as expected, except for STI index which is slightly smaller than others.

AR(1)-GJR(1,1) model given by (6)-(8) is estimated using three different density functions $U(0,1)$ for $\epsilon_i$: the Normal, the student’s-t, the Skew-t, see [16] for detail. After fitting the marginal distributions, the most appropriate copula function to express the bivariate distribution between JKSE and each of other four indexes is chosen. We analyses the tail dependence using copulas and observe the behavior of the dependence parameters in time-varying over the period of trading days. We chose skew-t for the distribution of the density functions as for $\epsilon_i$. This is due to the fact that the data show negative skewness and excess kurtosis.

As is indicated in Table 2, information criteria (AIC and BIC) and negative log-likelihood (LL) is used to select the copula function that is the best fit the pairs of index data. Table 2 shows that SJC copula is the best fit the pair JKSE-STI, while Normal copula fits better the rest of the pairs. These results still make sense, since the sample is relatively large, (1041 records). This result is in contradictions with the result found by [15], reporting that Normal copula is the worst fit among the copulas. However, they used ARMA(1,1)-GARCH(1,1)-skew-t, instead of AR(1)-GJR(1,1)-skew-t, to estimate the margins. Similar result was reported in [17] that SJC copulas enable risk managers to measure the risk more effectively than other copula functions.

Another tool that can be used to choose which copula could be best fit is test of goodness-of-fit (GoF). Some of these kinds of tests have been implemented in the literature to allow the model choice based on copula functions see for example in [15], [17], or [18]. The drawback of using AIC, BIC and LL is that those criteria reveal nothing about how good is the model fits to the data. They only give the order from the worst to the best one. Thus, the problem of deciding which of these copulas is the best “guess” for the data generating process remains unclear. In other words, it is necessary to conduct a more detailed assessment to make our final choice. In this paper, we do not conduct any further test to observe how good SJC in estimating tail dependence between JKSE and STI. Readers who are interested in further test for such a problem may refer to [18].

Another method that can be implemented to test how good the copula estimations are by carrying parametric simulation, proposed in [19]. The approach is started from choosing the candidate model, then estimating the parameters of the candidate model by maximum likelihood. Next, simulations are

Table 1. Log-Return Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>NIKEI</th>
<th>STI</th>
<th>HSENG</th>
<th>JKSE</th>
<th>KOSPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.000441</td>
<td>0.000167</td>
<td>0.000157</td>
<td>0.000545</td>
<td>0.000179</td>
</tr>
<tr>
<td>Median</td>
<td>0.0</td>
<td>0.1392</td>
<td>0.0</td>
<td>0.5410</td>
<td>0.0</td>
</tr>
<tr>
<td>Max</td>
<td>0.0552</td>
<td>0.0329</td>
<td>0.0552</td>
<td>0.0465</td>
<td>0.0490</td>
</tr>
<tr>
<td>Min</td>
<td>-0.1115</td>
<td>-0.0377</td>
<td>-0.0583</td>
<td>-0.0930</td>
<td>-0.0642</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0137</td>
<td>0.0079</td>
<td>0.0115</td>
<td>0.0465</td>
<td>0.0107</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.8711</td>
<td>-0.4003</td>
<td>-0.2743</td>
<td>-0.9004</td>
<td>-0.4207</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>9.0276</td>
<td>5.4957</td>
<td>5.9530</td>
<td>9.5897</td>
<td>7.3141</td>
</tr>
<tr>
<td>J-Bera</td>
<td>17059</td>
<td>297.68</td>
<td>390.91</td>
<td>20222</td>
<td>837.158</td>
</tr>
<tr>
<td># of obsv.</td>
<td>1041</td>
<td>1041</td>
<td>1041</td>
<td>1041</td>
<td>1041</td>
</tr>
</tbody>
</table>

As is indicated in Table 2, information criteria (AIC and BIC) and negative log-likelihood (LL) is used to select the copula function that is the best fit the pairs of index data. Table 2 shows that SJC copula is the best fit the pair JKSE-STI, while Normal copula fits better the rest of the pairs. These results still make sense, since the sample is relatively large, (1041 records). This result is in contradictions with the result found by [15], reporting that Normal copula is the worst fit among the copulas. However, they used ARMA(1,1)-GARCH(1,1)-skew-t, instead of AR(1)-GJR(1,1)-skew-t, to estimate the margins. Similar result was reported in [17] that SJC copulas enable risk managers to measure the risk more effectively than other copula functions.
carried out using estimated parameter of the candidate models. In next step, the model based on simulated data is estimated and the log-likelihood estimates are computed. Finally, the original log-likelihood estimates and the simulated log-likelihood estimates are compared. The models with good fits are those whose log-likelihoods in the simulated data are close to the original log-likelihoods.

### Table 2. Result for copula estimates with skew-t margins.

<table>
<thead>
<tr>
<th>Index pair</th>
<th>Dynamic Copula</th>
<th>ω</th>
<th>β</th>
<th>α</th>
<th>LL</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>JKSE and NIKKEI</td>
<td>Normal</td>
<td>0.7862</td>
<td>0.5147</td>
<td>-0.4406</td>
<td>-86.7433</td>
<td>-173.4809</td>
<td>-173.4666</td>
</tr>
<tr>
<td></td>
<td>RGC</td>
<td>1.9590</td>
<td>-0.9372</td>
<td>-1.3682</td>
<td>-79.4735</td>
<td>-158.9412</td>
<td>-158.9270</td>
</tr>
<tr>
<td></td>
<td>SJC($r^{(\pi)}$)</td>
<td>0.8051</td>
<td>-6.3459</td>
<td>-1.9483</td>
<td>-2.2469</td>
<td>-83.1910</td>
<td>-166.3705</td>
</tr>
<tr>
<td>JKSE and STI</td>
<td>Normal</td>
<td>-0.1228</td>
<td>0.0584</td>
<td>2.4037</td>
<td>-168.4965</td>
<td>-336.9332</td>
<td>-336.9189</td>
</tr>
<tr>
<td></td>
<td>RGC</td>
<td>1.7119</td>
<td>-0.4233</td>
<td>-2.2713</td>
<td>-134.8464</td>
<td>-340.5763</td>
<td>-340.5621</td>
</tr>
<tr>
<td></td>
<td>SJC($r^{(\pi)}$)</td>
<td>2.9280</td>
<td>-14.7227</td>
<td>-1.6617</td>
<td>-176.3833</td>
<td>-352.7552</td>
<td>-352.7266</td>
</tr>
<tr>
<td>JKSE and Hseng</td>
<td>Normal</td>
<td>1.5616</td>
<td>0.8474</td>
<td>-1.6835</td>
<td>-172.2593</td>
<td>-344.5129</td>
<td>-344.4986</td>
</tr>
<tr>
<td></td>
<td>RGC</td>
<td>1.0954</td>
<td>-0.0819</td>
<td>-1.7466</td>
<td>-163.0087</td>
<td>-326.0117</td>
<td>-325.9974</td>
</tr>
<tr>
<td>JKSE and KOSPI</td>
<td>Normal</td>
<td>-0.0108</td>
<td>0.0977</td>
<td>2.0770</td>
<td>-125.2798</td>
<td>-250.5539</td>
<td>-250.5396</td>
</tr>
<tr>
<td></td>
<td>RGC</td>
<td>1.9005</td>
<td>-0.7806</td>
<td>-1.6184</td>
<td>-116.0627</td>
<td>-232.1196</td>
<td>-232.1053</td>
</tr>
<tr>
<td></td>
<td>SJC($r^{(\pi)}$)</td>
<td>0.1371</td>
<td>-0.0149</td>
<td>0.5532</td>
<td>-0.0426</td>
<td>-119.0173</td>
<td>-238.0231</td>
</tr>
<tr>
<td></td>
<td>SJC($r^{(\pi)}$)</td>
<td>-1.9627</td>
<td>-1.1272</td>
<td>-0.0426</td>
<td>-119.0173</td>
<td>-238.0231</td>
<td>-237.9945</td>
</tr>
</tbody>
</table>

Finally, we analyze the tail dependence of the four index pairs. Figure 1 shows that the time dynamic of the dependence parameters of the copulas on the pair JKSE-NIKKEI. Figure 1 (top) shows the tail dependence dynamics for the Normal copula with symmetric tail dependence. The dependence coefficient for the Normal copula (constant) is 0.3868. The dynamic of this parameter over time is rather fluctuated. There are no periods that should be highlighted, because the fluctuation is a little over time. In Figure 1 (bottom) the dependence parameter for the RGC copula (lower tail dependence) with constant coefficient of 1.3036. The dynamic of this parameter over time is around this constant most of the observed sample. Moreover, this behavior shows a stable fluctuation over time.

The most general property of SJC copula is that it allows capturing the lower and upper tail dependence, as shown by Figure 2. The coefficient of lower tail dependence (constant SJC) is 0.2725 and the upper tail coefficient is 0.3642. The dynamic of the upper tail fluctuates around this value with very high frequency. This kind of characteristic is very common in financial market, i.e. the dependence on the higher tail is usually very volatile and not very informative.

The similar results are also found in pair between JKSE and Hang Seng and JKSE and KOSPI. We do not provide the figure showing the tail dependence behavior between JKSE and Hang Seng and JKSE and KOSPI due to the space limitations.

It is also important to note that the degree of association between JKSE and the other indexes seem to have constant evolution time, a moderate interconnection among these market except for dependence between JKSE and NIKKEI modeled by SJC copula which giving a very irregular in motions. To complete our analysis in the time evolution of the dependence structure, we evaluate the performance of each copula in terms of VaR, in the rest of this discussion.
To calculate Value at Risk (VaR) and Conditional VaR (CVaR), we choose the portfolio weight is 0.5 (for simplicity), which means that the portfolio is proportioned by 50% each index, mathematically, expressed as $r_p = 0.5r_{1,t} + 0.5r_{2,t}$. We use AR(1)-GJR(1,1) to estimate each margin and Student-$t$ for the innovation distribution. The VaR and CVaR analysis are reported in Table 3.
### Table 3. Estimated portfolio return constructed by JKSE and STI index and its VaR produced by SJC, RGC and Normal Copulas and violations at 5%-level.

<table>
<thead>
<tr>
<th>Index Pair</th>
<th>5%</th>
<th>1%</th>
<th>Violations</th>
<th>5%</th>
<th>1%</th>
<th>Violations</th>
<th>5%</th>
<th>1%</th>
<th>Violations</th>
<th>5%</th>
<th>1%</th>
<th>Violations</th>
</tr>
</thead>
<tbody>
<tr>
<td>JKSE-NIKKEI</td>
<td>1.71%</td>
<td>1.41%</td>
<td>21</td>
<td>1.43%</td>
<td>2.61%</td>
<td>25</td>
<td>1.31%</td>
<td>2.14%</td>
<td>21</td>
<td>2.61%</td>
<td>2.31%</td>
<td>25</td>
</tr>
<tr>
<td>JKSE-STI</td>
<td>1.45%</td>
<td>1.50%</td>
<td>25</td>
<td>1.51%</td>
<td>2.22%</td>
<td>25</td>
<td>2.55%</td>
<td>2.30%</td>
<td>27</td>
<td>2.66%</td>
<td>2.48%</td>
<td>27</td>
</tr>
<tr>
<td>JKSE-HSENG</td>
<td>1.72%</td>
<td>1.45%</td>
<td>16</td>
<td>1.51%</td>
<td>2.61%</td>
<td>25</td>
<td>2.55%</td>
<td>2.69%</td>
<td>16</td>
<td>2.66%</td>
<td>2.55%</td>
<td>27</td>
</tr>
<tr>
<td>JKSE-KOSPI</td>
<td>1.58%</td>
<td>1.43%</td>
<td>18</td>
<td>1.43%</td>
<td>2.30%</td>
<td>18</td>
<td>2.49%</td>
<td>2.70%</td>
<td>18</td>
<td>2.90%</td>
<td>2.90%</td>
<td>18</td>
</tr>
</tbody>
</table>

In the experiment for computing VaR, we simulate 2000 replications for each portfolio and for each of the four portfolios. As seen from Table 3 that Normal copula estimates VaR higher than RGC and SJC copula for all portfolios at the significant level of 1%. This means that VaR using Normal copula is more aggressive than RGC and SJC copula or in other words VaR portfolio estimated by Normal copula gives higher opportunities to investors to gain higher return as they set at a higher risk. This result is consistent with that quoted in [15], "it is sufficient to obtain good VaR estimates with a constant Normal copula".

In our backtesting experiment, we choose the first 700 data return as the sample for predicting VaR within 341 time window of observations. It is seen from Table 3 that exceedance occurs 25 times within 341 observations when backtesting VaR for JKSE-STI using Normal copula. This gives the probability of exceeding the VaR is \( P(X < -VaR_0) = 25/341 = 0.073 \) which is significant results at \( \alpha = 0.05 \), null hypothesis is rejected. The same results also occur for portfolio JKSE-NIKKEI for Normal, RGC and SJC copulas. The violations for the three copulas functions, Normal, RGC, and SJC are not significant at \( \alpha = 0.05 \). Overall, from the point of view of violations, Normal copula gives lower or the same exceedance than RGC and SJC copula for all portfolios, see Figure 3 for detail.

### 9. FINAL REMARKS

In this paper we estimate copula functions to capture the dependence structure and its time dynamic between JKSE index and NIKKEI, STI, HENGSENG, and KOSPI. The analysis of the dependence structure is started by fitting each margin with AR(1)-GJR(1,1) using three different density functions \( U(0,1) \) for the innovations \( \varepsilon_t \): the Normal, the student’s-t, the skewed-t distribution. The next step, using the estimates of the margins, we fit the three different time-varying copula functions, Normal, Rotated Gumbel, and SJC (Symmetrized Joe-Clayton) copulas, to estimate the joint distribution of the index pairs and observe the behavior of their dependence structures. The most appropriate copula is chosen based on the AIC, BIC and LL. Our results show that SJC copula is the best fit the pair JKSE-STI, while Normal copula fits better the rest of the pairs. Finally, we evaluate the impact of different copula on VaR estimation. We calculate VaR and CVaR from simulations of the conditional joint distribution derived from the fitted copulas, and conclude that VaR using Normal copula is more aggressive than RGC and SJC copula at significant level \( \alpha = 0.05 \) or in other words VaR portfolio...
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estimated by Normal copula gives higher opportunities to investors to gain higher return as they set at a higher risk. Backtesting was also carried out to test the accuracy of VaR and CVaR models.

Figure 3. Estimated portfolio return constructed by JKSE and STI index and its VaR produced by SJC, RGC and Normal Copulas

10. REFERENCE


